7.4 ~ The Quadratic Formula

Daily Objectives:

1. Use vertex form of a quadratic formula to find roots.
2. Derive the quadratic formula by completing the square.
3. Use the quadratic formula to solve application problems.

Example 1: Given the equation $0 = ax^2 + bx + c$, we will derive the quadratic formula to find the zeros of this equation.

$ax^2 + bx + c = 0$	Original equation.
$ax^2 + bx = -c$	Subtract c from both sides.
$a\left(x^2 + \frac{b}{a}x + \underline{\ ?\ }\right) = -c$	Factor to get the leading coefficient 1.
$a\left[x + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] = a\left(\frac{b}{2a}\right)^2 - c$	Complete the square.
$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$	Factor the perfect-square trinomial on the left side and multiply on the right side.
$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - \frac{4ac}{4a}$	Rewrite the right side with a common denominator.
$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$	Add terms with a common denominator.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Divide both sides by a.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Take the square root of both sides.
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	Use the power of a quotient property to take the square roots of the numerator and denominator.
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	Subtract $\frac{b}{2a}$ from both sides.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Add terms with a common denominator.

The Quadratic Formula

Given a quadratic equation written in the form $ax^2 + bx + c = 0$, the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

NOTE: When using the quadratic formula, your quadratic must be equal to ZERO *because* we want to know when quadratic hits the y-axis (aka when y = 0).

Example 2: Given the quadratic function $f(x) = -16x^2 + 120x + 3$, find exact and approximate solutions.

$a = -16 \quad b = 120 \quad c = 3$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-120 \pm \sqrt{(120)^2 - 4(-16)(3)}}{2(-16)}$

$\frac{-120 \pm \sqrt{14400 + 192}}{-32}$

$x = \frac{-120 \pm \sqrt{14,592}}{-32}$

$x = \frac{-120 + \sqrt{14592}}{-32} \qquad x = \frac{-120 - \sqrt{14,592}}{-32}$

$x = .0249 \qquad x = 7.5249$

Example 3: Solve $3x^2 = 5x + 8$ for the exact and approximate solutions.

$3x^2 - 5x - 8 = 0$

$\frac{--5 \pm \sqrt{(-5)^2 - 4(3)(-8)}}{2(3)}$

$\frac{5 \pm \sqrt{25 + 96}}{6}$

$x = \frac{5 \pm \sqrt{121}}{6}$

$x = \frac{5 \pm 11}{6}$

$x = \frac{5+11}{6} \qquad x = \frac{5-11}{6}$

$x = \frac{16}{6} \qquad x = -\frac{6}{6}$

$\boxed{x = 2\frac{2}{3} \quad x = -1}$

Investigation: Salvador hits a baseball at a height of 3 feet and with an initial upward velocity of 88 feet per second.

a. Let x represent time in seconds after the ball is hit, and let y represent the height of the ball in feet. Write an equation that gives the height as a function of time.

$y = -16x^2 + 88x + 3$

b. Write an equation to find the times when the ball is 24 feet above the ground.

$24 = -16x^2 + 88x + 3$

c. Rewrite your equation from (b) in the form $0 = ax^2 + bx + c$, and then use the quadratic formula to solve.

$0 = -16x^2 + 88x - 21$

$x = \frac{-88 \pm \sqrt{(88)^2 - 4(-16)(-21)}}{2(-16)}$

$x = \frac{-88 \pm \sqrt{7744 - 1344}}{-32}$

$x = \frac{-88 \pm \sqrt{6400}}{-32}$

$x = \frac{-88+80}{-32}$ $x = \frac{1}{4}$

$x = \frac{-88-80}{-32} = \frac{-168}{-32}$ $x = 5\frac{1}{4}$

What is the real world meaning of each of your solutions? Why are there two solutions?

The ball is 24 feet above the ground at $\frac{1}{4}$ seconds (on the way up) and at $5\frac{1}{4}$ seconds (on the way down).

d. Find the y-coordinate of the vertex of this parabola. How many different x-values correspond to this y-value.

Average the zeros $x = \frac{\frac{1}{4} + 5\frac{1}{4}}{2}$

$x = \frac{5\frac{1}{2}}{2} = \frac{11}{2} \cdot \frac{1}{2} = \frac{11}{4}$

$y = -16\left(\frac{11}{4}\right)^2 + 88 \cdot \frac{11}{4} + 3$

$= -16\left(\frac{121}{16}\right) + 242 + 3$

$= -121 + 242 + 3$

$= 121 + 3$

$y = 124$

ONLY 1 X-VALUE

e. Write an equation to find the time when the ball reaches its maximum height. Use the quadratic formula to solve this equation.

$124 = -16x^2 + 88x + 3$

$0 = -16x^2 + 88x - 121$

$\frac{-88 \pm \sqrt{88^2 - 4(-16)(-121)}}{2(-16)}$

$\frac{-88 \pm \sqrt{7744 - 7744}}{-32}$

$\frac{-88 \pm 0}{-32}$

$x = \frac{-88}{-32} = \frac{11}{4} = 2\frac{3}{4}$

At what point in the solution process does it become obvious that there is only one solution?

± 0

f. Write an equation to find the time when the ball reaches a height of 200 feet. What happens when you try to solve using the quadratic formula?

$200 = -16x^2 + 88x + 3$

$0 = -16x^2 + 88x - 197$

$\frac{-88 \pm \sqrt{88^2 - 4(-16)(-197)}}{2(-16)}$

$= \frac{-88 \pm \sqrt{7744 - 12,608}}{-32}$

$= \frac{-88 \pm \sqrt{-4864}}{-32}$

NO SOLUTION

200 ABOVE MAX